RESEARCH NOTE

Multiple spatial representations of number: evidence for co-existing compressive and linear scales

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Abstract Although the spatial representation of number (mental number line) is well documented, the scaling associated with this representation is less clear. Sometimes people appear to rely on compressive scaling, and sometimes on linear scaling. Here we provide evidence for both compressive and linear representations on the same numerical bisection task, in which adult participants estimate (without calculating) the midpoint between two numbers. The same leftward bias (pseudoneglect) shown on physical line bisection appears on this task, and was previously shown to increase with the magnitude of bisected numbers, consistent with compressive scaling (Longo and Lourenco in Neuropsychologia 45:1400-1407, 2007). In the present study, participants held either small (1-9) or large (101–109) number primes in memory during bisection. When participants remembered small primes, bisection responses were consistent with compressive scaling. However, when they remembered large primes, responses were more consistent with linear scaling. These results show that compressive and linear representations may be accessed flexibly on the same task, depending on the numerical context.

Keywords Numerical cognition · Mental number line · Compressive and linear scaling

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Introduction

The spatial representation of number has been well established. Much evidence suggests that numbers are represented along a so called *mental number line*, oriented (at least in Western culture) with increasing values from left to right (e.g., Dehaene et al. 1993; Fischer et al. 2003; Loetscher et al. 2008). The scaling of numerical representation, however, is less clear. Although two types of scale compressive (e.g., Dehaene and Mehler 1992; Piazza et al. 2004) and linear (e.g., Gallistel and Gelman 1992, 2000) have been proposed, there is disagreement as to which type better depicts the spatial organization of number. Here we provide evidence for the co-existence of compressive and linear numerical scales, as well as insight into the dynamics that may support access to each type of scale.

Space and number

Perhaps the classic demonstration of the relation between space and number comes from experiments showing that parity (odd/even) judgments are faster for smaller numbers (e.g., 1 and 2) when executed in the left hemi-space, such as when using one's left hand, and for larger numbers (e.g., 8 and 9) when executed in the right hemi-space, such as when using one's right hand, the so called spatial-numerical association of response codes (SNARC) effect (e.g., Dehaene et al. 1993; Shaki and Fischer 2008). Spatial-numerical associations have also been demonstrated on bisection tasks. Patients with hemi-spatial neglect, which typically occurs following injury to right posterior parietal cortex and parieto-frontal connections in underlying white matter (e.g., Bartolomeo et al. 2007; Bisiach and Vallar 2000), tend to ignore the left side of space, indicating the midpoint of physical lines too far to the right. Some of these patients

show analogous effects when asked to 'bisect' numerical intervals, estimating (without calculating) the number midway between two others. Zorzi et al. (2002) found that these patients respond with numbers larger than the true midpoint, as if showing *rightward* bias along a mental number line (also, Zorzi et al. 2006; although, see, Doricchi et al. 2005). Recently, Pia et al. (2009) described a patient with right neglect following damage to the left posterior parietal cortex who showed leftward biases for both physical and mental number line bisection.

Numerical scaling

Dehaene et al. have argued that the mental number line is non-linearly compressive, such that the subjective space allocated to numbers becomes smaller with increasing numerical magnitude (e.g., Dehaene 2001; Dehaene and Mehler 1992; Piazza et al. 2004; also, Nieder and Miller 2003). In contrast, Gallistel et al. have argued that number is organized linearly, such that the subjective distance between numbers remains constant, albeit more variable, across magnitude (e.g., Gallistel and Gelman 1992, 2000; also, Brannon et al. 2001; Whalen et al. 1999). It has often been difficult to distinguish between these models, since they tend to make identical behavioral predictions, and, when they do make differential predictions, Western adults sometimes appear to rely on compressive scales (e.g., Banks and Coleman 1981; Banks and Hill 1974; Longo and Lourenco 2007; van Oeffelen and Vos 1982), and, on others, on linear scales (e.g., Banks and Coleman 1981; Dehaene et al. 2008; Siegler and Opfer 2003).

On the number bisection task described above, we (Longo and Lourenco 2007) found that, as in physical line bisection in which healthy adults generally show a slight leftward bias, known as *pseudoneglect* (Jewell and McCourt 2000), they also show leftward bias when 'bisecting' the interval between two numbers, underestimating the true midpoint. In addition, this bias increases with the magnitude of the numbers to be bisected, consistent with compressive scaling. Constant leftward attentional bias on this task leads to increasing leftward numerical bias because larger numbers are subjectively closer together (see Fig. 1, top). Previous studies have reported numerical modulation of spatial attention. Fischer et al. (2003), for example, showed that perceiving smaller versus larger numbers biased spatial attention leftward and rightward, respectively. Variation in spatial attention is not likely to account for the pattern of bisection responses, however. In the number bisection task, leftward bias increased with numerical magnitude, the opposite of what would be predicted if perceiving numbers affects spatial attention, suggesting that attentional bias is likely to be approximately constant on this task.

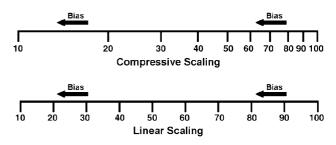


Fig. 1 The effects of leftward attentional bias (i.e., pseudoneglect) on the number bisection task, given compressive (*top*) versus linear (*bottom*) scaling of number. With compressive scaling, the extent of numerical bias (i.e., underestimation of the midpoint for numerical intervals) increases with greater numerical magnitude (of the midpoint). With linear scaling, the extent of numerical bias remains constant regardless of magnitude

Why might numerical representations appear compressive on some tasks and linear on others? One possibility is that number is actually represented with multiple scales, compressive and linear, which are used flexibly depending on the demands of the task. What demands might favor one scale over another? Dehaene et al. (2008) recently suggested that the (universal) default scale of number is compressive, with increasing reliance on linear representations driven by particular cultural experiences such as language and schooling. Consistent with this view are findings showing a developmental transition from compressive to linear scaling (Siegler and Opfer 2003; also, Booth and Siegler 2006; Siegler and Booth 2004), and variation in adults across culture, with linear scaling in Westerners and compressive scaling in the Mundurukú, an Amazonian population (Dehaene et al. 2008).

As discussed below, there are adaptive reasons for representing numerical information along compressive scales. One reason concerns the psychological significance of making errors when discriminating smaller numerical values versus larger values. It is frequently the case that differences at the lower end of the scale are more meaningful than those at the higher end (e.g., Nieder 2005). Relatedly, people tend to have more experience, and, hence, greater familiarity with smaller numerical values. As in cases where greater experience leads to changes in the allocation of representational resources (e.g., Elbert et al. 1995), more exposure to smaller numbers might lead to their (spatial) over-representation via compressive scaling. Particularly important for supporting access to linearly scaled representations, then, may be exposure to large numbers. Indeed, Siegler and colleagues (e.g., Siegler and Booth 2004; Siegler and Opfer 2003) have suggested that greater overall experience with small numbers, especially earlier in life, might account for the initial reliance on compressive scaling, wherein greater representational space is allocated to more familiar numerical values.

Present study

The purpose of the present study was twofold: (1) to test whether Western adults have access to both compressive and linear scales on the same task, and (2) to test the conditions that mediate access to the different scales. If number is represented with both types of scale, it may be possible to prime their use, differentially, on the same task. We tested participants under different memory conditions (maintenance of small versus large numbers) on our number bisection task, in which participants have been shown to rely, by default, on compressive scaling (Longo and Lourenco 2007). Compressive scales have the effect of over-representing small numbers, whereas linear scales give equal representational weight to small and large numbers. Thus, maintaining larger numbers in memory, which would have the effect of making these numbers more salient than is typically the case, and, hence, more familiar, should result in greater reliance on linear scaling. Conversely, maintaining smaller numbers in memory should reinforce the use of compressive scaling. On this number bisection task, linear scaling should lead to consistent leftward numerical bias across magnitude since the subjective spacing between numbers does not vary (see Fig. 1, bottom); this contrasts with compressive scaling in which numerical bias increases (i.e., shifts even more leftward) with increasing magnitude.

Method

Participants

Fifteen students (11 females) between 18 and 23 years (M = 19.27, SD = 1.67) participated for course credit or payment (\$10). The majority were right-handed (N = 12, M = 51.7, SD = 68.1), as measured by the Edinburgh inventory (Oldfield 1971). Experimental procedures were approved by the local ethics committee.

Stimuli, design, and procedure

Participants sat approximately 55 cm from a 17 in. (43.2 cm) computer monitor. Number pairs $(1.25^{\circ}$ in height) were presented using Matlab (MathWorks, Natick, MA, USA) script, centered on the screen, and separated by a small horizontal line. Numbers varied between 11 and 99, randomly selected. The same 216 pairs were used for each participant. Smaller numbers in these pairs ranged from 11 to 85 with a mean of 35.97 (SD = 18.39) across all instances. Larger numbers in these pairs ranged from 23 to 99 with a mean of 74.02 (SD = 19.02) across all instances. By using a wide range of numbers, we would be able to test for differences in the magnitude of the number pairs and

interval size. Based on previous work showing ceiling effects for smaller intervals of number pairs (e.g., Longo and Lourenco 2007; Zorzi et al. 2002), intervals here ranged from 11 to 87 (M = 38.72, SD = 1.26).

Participants estimated the number midway between each pair of numbers. They were told not to compute the answer, but to answer as quickly as they possibly could, using whichever number seemed immediately intuitive. Prior to the presentation of number pairs, participants were primed with three different numbers, presented sequentially, at the top, bottom, and center of the screen. Each number was presented for 500 ms, with the order (top, bottom, center) randomly determined on each trial. Participants were asked to recall the three prime numbers after indicating their bisection response. On half the trials, participants were presented with small primes (1-9), and, on the other half, with large primes (101–109); prime numbers on each trial were randomly selected. We used primes outside the range of the number pairs presented for bisection stimuli for two reasons: (1) to highlight the 'smallness' and 'largeness' of the primes, and (2) to avoid any direct memory interference between the primes and the bisection stimuli. The experiment was divided into six blocks of 36 trials, with each block comprised of 18 trials of small and large primes. On half the trials in each block, the smaller number in the pairs to be bisected appeared on the left, and, on the other half, on the right. Trial order was randomized. Responses were verbal, and recorded by an experimenter who was seated behind the participant.

Results

All participants made errors in reporting the primes (M = 9.48%, range 1.8-27.78%). Approximately half the errors involved remembering small primes as large primes (M = 53.85%, SD = 24.68%), t(14) = 0.60, P > 0.1. Because of these errors, analyses were conducted on trials as a function of remembered primes. Trials on which bisection responses were outside the interval of number pairs were excluded from the analyses (M = 1.9%, range 0-11.11%).

For each number pair, deviation scores were computed by subtracting the true midpoint (i.e., arithmetic mean) from participants' bisection responses. Significant underestimation of the midpoint, that is, leftward bias was observed for both conditions (small primes: M = -2.10, SD = 1.76, t(14) = -4.62, P < 0.001; large primes: M = -1.29, SD = 1.58, t(14) = -3.16, P < 0.01), whether the smaller number in the pair was presented on the left or right (all *P* values < 0.05). For both conditions, the majority of participants showed overall leftward bias in their bisection responses (small primes: 14/15; large primes: 14/15; both *P* values < 0.001, binomial test). Effects of priming and numerical magnitude

Change in bias with numerical magnitude was investigated using least-squares regression to compute slopes for each participant in each condition regressing bias on the mean of the numbers to be bisected. In the small primes condition, regression slopes were significantly negative, $\beta = -0.049$, t(14) = -7.39, P < 0.0001 (see Fig. 2, top), indicating that leftward bias increased as numerical magnitude increased. This suggests that participants relied on compressive scaling, as in previous research with no priming (Longo and Lourenco 2007). Similar effects were observed with the smaller number in the pairs on the left, $\beta = -0.052$, t(14) = -5.98, P > 0.0001, or right, $\beta = -0.047$, t(14) = -5.19, P < 0.0001.

In contrast, in the large primes condition, regression slopes did not differ significantly from zero, $\beta = -0.013$, t(14) = -1.53, P > 0.1 (see Fig. 2, bottom). Similar effects were observed with the smaller number on the left, $\beta = -0.020$, t(14) = -1.98, P > 0.06, or right, $\beta = -0.006$, t(14) = -0.489, P > 0.1. Additionally, regression slopes in the large primes condition differed significantly from those in the small primes condition, t(14) = 3.79, P < 0.01, d = 1.21, with the majority of participants showing reduced

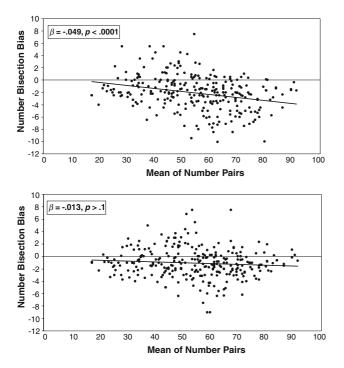


Fig. 2 Numerical bias as a function of numerical magnitude (calculated as the mean of the two numbers in a pair) for remembered small primes (*top*) and large primes (*bottom*) conditions. In the small primes condition, bias increased with numerical magnitude, suggesting that participants relied on compressive scaling during number bisection. In the large primes condition, bias remained relatively constant across numerical magnitude, suggesting that participants relied on linear scaling

slopes (13/15, P < 0.05, binomial test). These results suggest that participants relied on linear scaling during number bisection on trials in which they held large number primes in memory.

Could the difference between conditions result from a more general increase in the numerical values of bisection responses? Having been primed with large numbers, participants might have over-estimated the midpoint regardless of magnitude. Although the reduction in slope argues against this possibility, since greater numerical bisection responses would not predict a change in slope, it is worth noting that for both conditions the extent of bias was comparable for smaller number pairs. That is, analyses comparing the lower quartile of number pairs revealed no significant difference in bias between small primes (M = -1.09, SD = 2.33) and large primes (M = -0.76,SD = 1.85) conditions, t(53) = -0.92, P > 0.1, suggesting that greater overall numerical responses does not account for the change in slope. Another possible explanation for the difference between conditions concerns numerical interval. Siegler and Opfer (2003) showed that, at least in young children, a smaller numerical interval invoked linear scaling, whereas a larger interval invoked compressive scaling (see, also, Banks and Coleman 1981). As in Longo and Lourenco (2007), although overall *error* for each participant increased significantly with increasing interval size in small primes (mean r = 0.39), t(14) = 13.06, P < 0.0001, and large primes (mean r = 0.42), t(14) = 15.27, P < 0.0001, conditions, there was no significant increase in directional bias for each participant with increasing interval size in either condition (both P values > 0.1). This suggests that the difference in slope across the two conditions was not driven by effects of interval size, but, rather, by exposure to small versus larger number priming.

The analyses above were conducted on remembered primes (i.e., the prime numbers participants actually reported seeing). We also conducted separate analyses on the presented primes (i.e., the prime numbers that appeared on the computer monitor on each trial). When recall was not factored into the regression analyses, regression slopes were significantly negative in both small primes, $\beta = -0.036$, t(14) = -3.86, P < 0.01, and large primes, $\beta = -0.026$, t(14) = -3.69, P < 0.01, conditions, which did not significantly differ, t(14) = -0.96, P > 0.1. In other words, the change in slope observed in the large prime condition only occurred if participants remembered the primes as larger numbers. That there was no difference when recall was not factored into the analyses suggests that active maintenance of-rather than merely passive exposure tosmall versus large number primes was critical to determining reliance on compressive versus linear scaling.

Could differences between the two conditions be due to differential working memory demands? Although Doricchi

et al. (2005) have pointed to a relation between (spatial) working memory and number bisection responses in patients with hemi-spatial neglect, there are reasons to believe that the present results with healthy adults are not due to different memory demands. First, the number of recall errors did not differ between the two conditions (small primes condition: M = 12.47, SD = 12.69; large primes condition: M = 8.00, SD = 4.12; t(14) = 1.34, P > 0.1), suggesting that working memory demands did not in fact differ across conditions. Furthermore, if anything, greater working memory demands would be predicted in the large primes condition, which appeared to lead to more linear scaling. Given that the default numerical representation appears to be compressive (Dehaene et al. 2008; Longo and Lourenco 2007; Siegler and Opfer 2003), the higher load condition would be expected to lead to increased compression, the exact opposite of what was observed.

Discussion

The present findings demonstrate that the same bisection task can elicit compressive and linear representations of number in the same individuals, depending on the numerical context. When the context involved maintaining small number primes in memory, the leftward bias on number bisection increased with numerical magnitude, consistent with compressive scaling. When the context involved maintaining larger number primes in memory, the leftward bias remained relatively constant, consistent with linear scaling. In a previous study, with no priming conditions, participants relied on compressive scaling to bisect numerical intervals (Longo and Lourenco 2007). Although the apparent default on this task is compressive, the present findings show that Western adults have access to both compressive and linear representations, which are deployed flexibly on a single task.

Dehaene et al. (2008) recently suggested that the universal default representation of number is compressive, and that linear representation is a cultural invention, seen more commonly in Western than Indigenous cultures. They suggested that experiences related to measurement, and to addition and subtraction lead to the gradual development of linear scaling. Siegler and Opfer (2003) showed flexibility across development in Western children, with a shift from compressive to linear scaling on a task in which numbers were explicitly placed at particular locations along a line segment. Importantly, flexibility was also observed within a single age depending on the numerical context. Specifically, second-graders' placement of numbers varied as a function of the interval marking the ends of the line. With the smaller interval (0–100), children distributed the numbers to be placed on the line evenly, consistent with linear scaling. However, with the larger interval (0–1,000), they allocated more space to the smaller numbers (e.g., placing 25 near the middle of the line), consistent with compressive scaling. That responses depended on the numerical interval suggests that greater familiarity with larger numbers may be an important factor in supporting access to linearlyscaled representations. The present results dovetail with these findings by showing that both compressive and linear representations of number co-exist, and that this holds for adults as well as children, across different tasks.

Although multiple representations of number might appear inefficient, lacking neural economy (e.g., Dehaene 2008), co-existing compressive and linear scales make a great deal of adaptive sense, especially since each type might be better suited to particular task dynamics. Thus, the default numerical scale on a given task would depend on the relative advantage of that scale for that task. For example, compressive scaling might be advantageous when exact distinctions for small numbers are critical (e.g., Dehaene 1997; Nieder 2005), which, as discussed above, may be the more common scenario. In general, errors in precision are more likely to impact behaviors involving smaller numerical values than those involving larger values. The ecological salience of encountering two predators versus one predator, for example, would be greater than encountering 20 versus 19. In the former case, there might be the option to fight or flee; in the latter, the best option would almost certainly be flight. Linear scaling, in contrast, provides a more veridical description of the actual state of the world. The linear representation of number might be particularly advantageous when precise discriminations are also necessary for larger numerical values (e.g., Gallistel and Gelman 2000) where compressive scaling would most certainly lead to biased judgments. Precise discriminations with larger numerical values may be particularly critical when errors of even a single unit could have serious consequences, as when determining one's tax bracket.

Our results suggest that greater active experience with larger numbers may highlight the need for making precise distinctions with these values. Although cultural and developmental factors, noted above, may exert their own influence, exposure to larger numbers is likely to co-vary with these factors. Recent findings have demonstrated cultural effects on numerical scaling, with differences between Western adults and an indigenous population known as the Mundurukú (Dehaene et al. 2008). Our findings suggest that similar differences may occur even within Western adults as a function of using large numbers, and, perhaps, other numerical-related expertise.

A large body of research has demonstrated that representations of number are inherently spatial, organized along a *mental number line* from left to right. The scale of this number line, however, has been controversial, and two types have been proposed: linear and compressive. Although both types provide attractive models of numerical representation, it has been difficult to distinguish between them given that some data appear more consistent with linear scaling and other data with compressive scaling. The present study sheds light on this controversy by providing evidence for the coexistence of both types of numerical representations in Western adults. Although our data speak clearly to the use of multiple spatial representations of number, they do not address specific questions concerning the underlying dynamics of these representations. Are there separate static compressive and linear representations of number, or do these representations emerge on-line as a function of the task demands? These are important questions for future research.

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